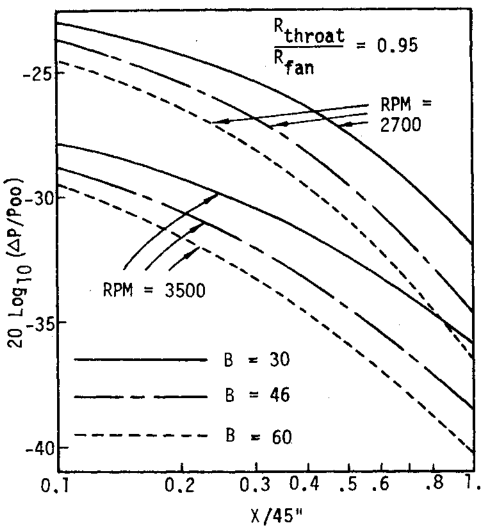

Fig. 2 Effects of blade spacing on shock decay; stagger angle = 70 deg.

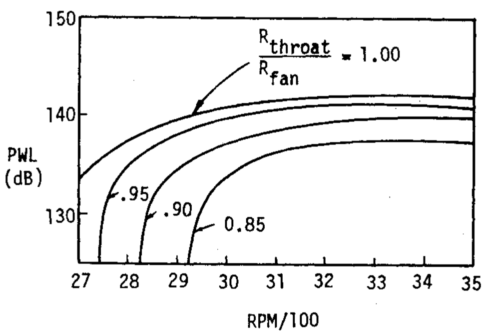

Fig. 3 Effects of inlet contour and power setting on PWL at inlet opening; $B = 46$.

shows that there is a power setting corresponding to a maximum PWL for a given inlet contour and blade number; the P_{00} in Fig. 2 is the atmospheric pressure at sea level and 59°F.

Figure 3 shows that the total $(PWL)_x$ at the throat station will increase sharply when the supersonic fan noise is just cut-on, and then changes slightly with further increasing power setting.

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Direct and Inverse Calculation of the Laminar Boundary-Layer Solution

Pascal L. Ardonceau*
and
Thierry Alziary de Roquefort†
Centre d'Etudes Aerodynamiques et Thermiques, Poitiers, France

Introduction

ANALYTICAL and numerical investigations on the boundary-layer equations have demonstrated that the solution becomes singular at the separation point when the pressure is specified as a consequence of the matching procedure with the outer flow.[1-4] In order to overcome this difficulty an inverse formulation (displacement thickness or specified skin friction) has been successfully applied to obtain regular solutions of the boundary-layer equations for slightly or moderately separated flows.[5,6] The aim of this paper is to present the relationship existing between two typical parameters (pressure and displacement thickness) at some specified longitudinal location. Results are computed for various states of the incoming boundary-layer. From the possible variation range of the specified quantity [$p(x)$ or $\delta^*(x)$], the opportunity to solve the direct or inverse problem may be evaluated. It is shown that the existence of δ^* specified boundary-layer solutions is nontrivial for accelerated flows, which is the counterpart of the possibility to obtain regular solutions near or inside separated regions.

Numerical Integration of Boundary-Layer Equations

The present work was initiated in order to calculate the laminar boundary-layer/shock-wave interaction, including separation effects. The boundary-layer equations are solved simultaneously with the external inviscid flow in an iterative manner.[3-7] The computer program is written in such a form that either direct or inverse boundary-layer solution may be calculated at each X step. Assuming a Prandtl number of 1, to delete the energy equation, the set of equations may be written

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho u}{\partial y} = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{\partial(\mu \partial u / \partial y)}{\partial y} \qquad (2)$$

$$p/\rho = Rh/Cp \qquad (3)$$

$$h + u^2/2 = h_T = Cte \qquad (4)$$

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*Research Assistant, Centre National de la Recherche Scientifique.

†Associate Professor, Mecanique des fluides, Universite de Poitiers.

In addition to the usual boundary wall conditions an additional outer condition is required:

1) Direct solution: $dp/dx = (dp/dx)$ specified.

2) Inverse solution:

$$\int_0^\infty (1-\rho u/\rho_e u_e)\,dy = [\delta^*(x)] \text{ specified.}$$

The equations are written in finite-difference form and linearized (second-order X and Y). At each X_j step the momentum equation is solved alone, resulting in a tridiagonal matrix for specified pressure or tridiagonal +1 row and 1 column for specified displacement thickness. The Thomas algorithm[5] is applied and the other equations (continuity, state, and energy) are solved. The process is repeated until convergence and then the X_{J+1} step is calculated. As a first guess the X convection terms are set equal to zero in the separation bubble where $U(X,Y)<0$, in order to maintain the $+X$ parabolic nature of the boundary-layer equations.[8] In fact, the calculation of the separation region requires several X sweeps with a centered or forward difference scheme in the separated region to deal with downstream influence.[5,6] Comparative tests show that neglecting the X convection terms does not lead to large errors.

The simultaneous solution of the boundary-layer equations and of the external flowfield through several iterations requires calculation of "nonphysical" boundary-layer flows characterized by some $p(X)$ or $\delta^*(X)$ distribution.

In order to know what condition may be imposed on the boundary-layer solution the following procedure is applied:

1) The boundary-layer is calculated up to the point X_J, where the boundary-layer pressure is $p(J)$ and the displacement thickness $\delta^*(J)$.

2) Various conditions are applied to the X_{J+1} boundary-layer profile using either direct or inverse calculation.

A relation between $p(J+1)$ and $\delta^*(J+1)$ is then obtained. Since the relation between $p(J+1)$ and $\delta^*(J+1)$ depends upon the mesh size Δx, it is more convenient to introduce a gradient representation:

$$\frac{dp}{dx}\left(J+\frac{1}{2}\right)=f\left[\frac{d\delta^*}{dx}\left(J+\frac{1}{2}\right)\right] \quad (5)$$

Equation (5) is approximated by $(p_{J+1}-p_J)/\Delta x = f[(\delta^*_{J+1}-\delta^*_J)/\Delta x]$ and is computed for a supersonic boundary layer ($M_\infty=2$, $Re_x \simeq 3.10^5$) which is initially accelerated or retarded and for the case $dp/dx=0$ up to the X_J location. See Figs. 1-3.

Discussion of Results

The curves

$$\frac{d\delta^*}{dx}\left(J+\frac{1}{2}\right)=f\left[\frac{X_{J+1}}{P_\infty}\frac{dp(J+\frac{1}{2})}{dx}\right]$$

exhibit a rather similar shape for the three computed cases. The initial condition, in other words the type of flow encountered by the boundary layer up to the J station (specified positive, zero, or negative pressure gradient), does not induce a fundamental modification of its shape but rather a translation in the grad p/θ plane ($\theta = d\delta^*/dx$ slope of the displacement surface).

Two typical regions may be distinguished: 1) a slight slope branch including negative and mildly positive pressure gradients, and 2) a second branch with quasi-infinite slope which corresponds to the boundary-layer separation.

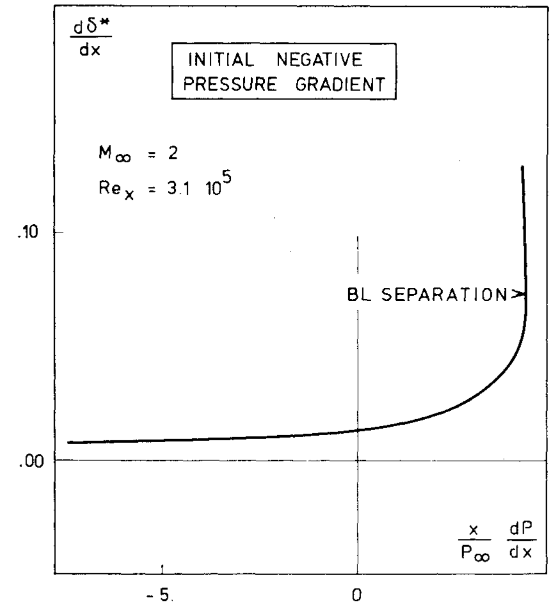

Fig. 1 Relation between $d\delta^*/dx$ and dp/dx for initial negative pressure gradient.

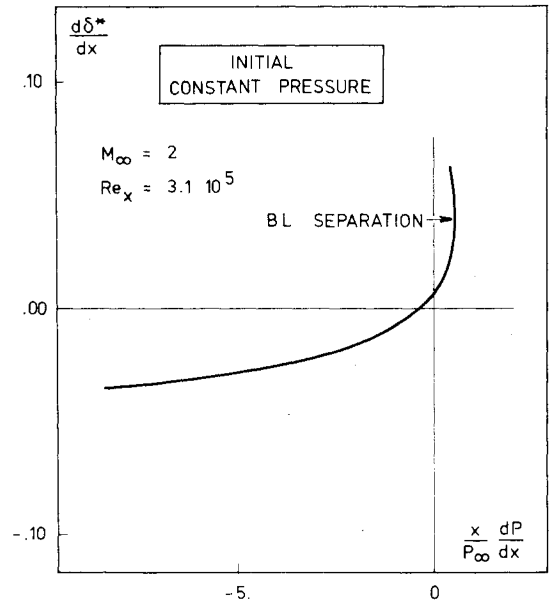

Fig. 2 Relation between $d\delta^*/dx$ and dp/dx for initial constant pressure gradient.

Figure 3 shows that the pressure gradient cannot be strong in separated flows. This is related to the constant pressure (plateau pressure) encountered in laminar shock-wave/boundary-layer interactions with flow separation.

From observation of the results, two important features may be deduced:

1) Near or inside separation the pressure gradient is always small and cannot exceed a maximum value which is close to zero. Moreover, the solution of the direct problem (prescribed pressure gradient) appears to be nonunique in the range $0 \le \text{grad } p \le \text{grad } p_{max}$. The failure of direct boundary-layer calculation at separation may be explained by the behavior of the derivative $d\theta/d$ grad p which is positive in attached flows, infinite at separation, and mildly negative inside separation. From the preceding considerations it may be deduced that an inverse calculation of the boundary layer is very well adapted to retarded flows and the solution $p(x)$ is very precisely defined by the imposed condition $\delta^*(x)$.

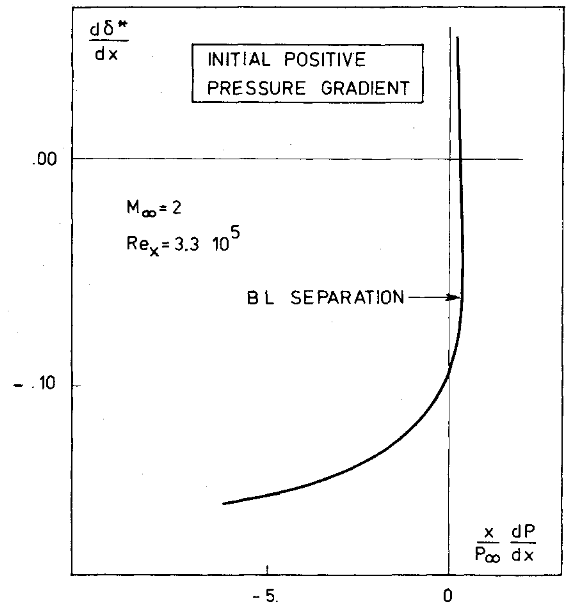

Fig. 3 Relation between $d\delta^*/dx$ and dp/dx for initial positive pressure gradient.

2) Conversely for flows with gradient $p<0$ the derivative d grad $p/d\theta$ becomes very large. Hence a small variation in the imposed condition $\theta(x)$ or $\delta(x)$ may induce large differences in the result and the computation may fail if θ is such that $dp/dx \rightarrow -\infty$. Thus a direct method of computation seems more convenient if the boundary layer is expected to accelerate.

The ultimate method of computation appears to be a "mixed" method where the applied condition is neither $\delta^*(x)$ nor $p(x)$ but a linear combination of the two. A convenient linear relation appears to be: $\delta^*(x)+Ap(x)=B$ where A is a function of the slope $d\delta^*/dp(x)$.

Conclusion

A relation between the quantities $p(x)$ and $\delta^*(x)$ which may be prescribed in a boundary-layer calculation by direct or inverse method of solution is computed. The typical aspect of the curve $d\delta^*/dx=f(dp/dx)$ strongly supports the fact that inverse methods should be used to compute retarded or separated flows, but that direct method of solution is highly preferable for negative pressure gradients. A "mixed," inverse/direct method is now applied to calculate boundary-layer flows in various conditions.

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Turbulent Boundary-Layer Calculations in Adverse Pressure Gradient Flows

C. E. Jobe* and W. L. Hankey†
Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio

Introduction

ALGEBRAIC eddy viscosity models are known to be inadequate for computing flows with adverse pressure gradients, as found on airfoils near the trailing edge. It may be some time before higher-order turbulence models are more accurate and reliable than this simpler model for engineering purposes, and the computational resources required for the use of higher-order models often prohibit their use for all but relatively simple geometries. Thus, this investigation was undertaken to determine if the range of usefulness of the two-layer algebraic eddy viscosity model could be extended to supersonic flows nearing separation. Numerical solutions of the turbulent boundary-layer equations are compared with the experimental data of Laderman,[1] Sturek,[2] and Waltrup and Schetz,[3] with the objective of increasing the accuracy of the two-layer eddy viscosity model for flows encountering constant, adverse pressure gradients by adjusting the three constants therein to obtain agreement with experiment.

Numerical Procedure

The compressible, turbulent boundary-layer equations were cast in nondimensional form by Shang et al.,[4] using the algebraic eddy viscosity model with its extension to the apparent heat flux term and the Levy-Lees transformation. Three multilayer eddy viscosity models were evaluated by these authors for zero pressure gradient, adiabatic flows over a range of Mach numbers, and Reynolds numbers. They found that the Cebeci et al.[5] eddy viscosity model was more accurate over a wide range of flow conditions. The details of this method are contained in Ref. 4.

In this model, the equivalent eddy viscosity, ϵ, is written as $\nu+\Gamma\epsilon_i$ in the inner region, consisting of the viscous sublayer and wall regions, and as $\nu+\Gamma\epsilon_0$ in the outer wake region, where ν is the molecular kinematic viscosity. The function Γ represents the transition model of Dahawan and Narasimha as developed by Shang et al.,[4] and was zero for laminar flow and unity for fully turbulent flow. The present calculations were started with a laminar boundary-layer solution at the wind tunnel throat or boundary-layer origin, and transition was initiated when the length Reynolds number exceeded 0.5 million.

The eddy viscosity in the inner region was

$$\epsilon_i = k_1^2 y^2 D^2 \left| \frac{\partial u}{\partial y} \right|$$

where D is the Van Driest damping factor. In the outer region the model was

$$\epsilon_0 = k_2 u_e \delta_i^* \gamma(y/\delta)$$

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*Aerospace Engineer. Associate Fellow AIAA.
†Senior Scientist. Associate Fellow AIAA.